

# Pensans CP School



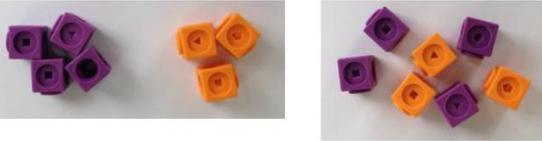
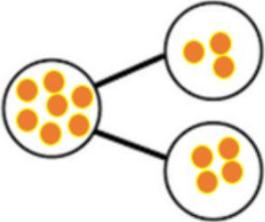
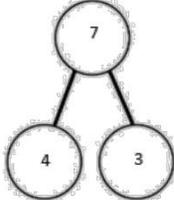
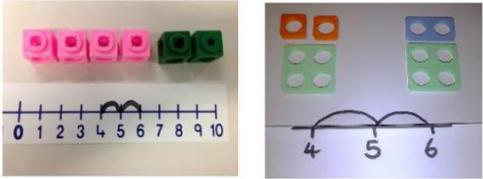
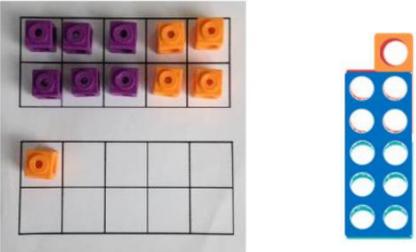
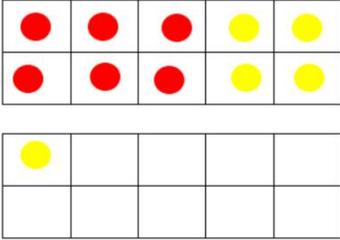
## Maths Calculation Policy

Reviewed on February 2017

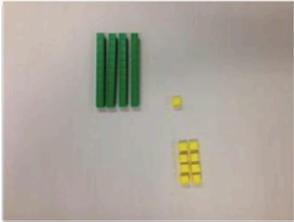
Next review date February 2019

# Addition-

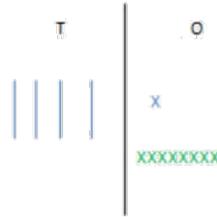
Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract
<p><b>Combining two parts to make a whole</b> (use other resources too e.g. eggs, shells, teddy bears etc)</p> 		<p><math>4 + 3 = 7</math> (four is a part, 3 is a part and the whole is seven)</p> 
<p><b>Counting on using number lines</b> by using cubes or numicon</p> 	<p>A bar model which encourages the children to count on</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? <math>4 + 2</math></p> 
<p><b>Regrouping to make 10</b> by using ten frames and counters/cubes or using numicon: <math>6 + 5</math></p> 	<p>Children to draw the ten frame and counters/cubes</p> 	<p>Children to develop an understanding of equality e.g <math>6 + \square = 11</math> and</p> <p><math>6 + 5 = 5 + \square</math> <math>6 + 5 = \square + 4</math></p>

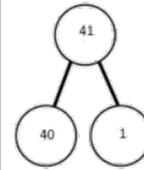
**TO + O using base 10.** Continue to develop understanding of partitioning and place value  $41 + 8$



Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.



$41 + 8$



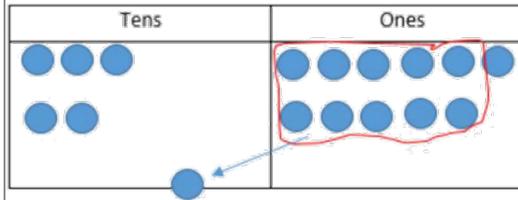
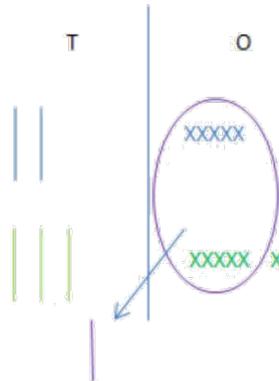
$1 + 8 = 9$   
 $40 + 9 = 49$

	4	1
+		8
<hr/>		
	4	9

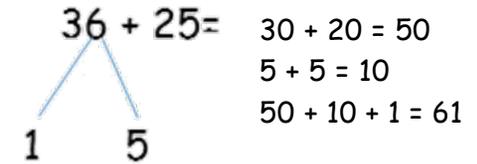
**TO + TO using base 10.** Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging.  $36 + 25$

	Tens	Ones
+		
=		

This could be done one of two ways:



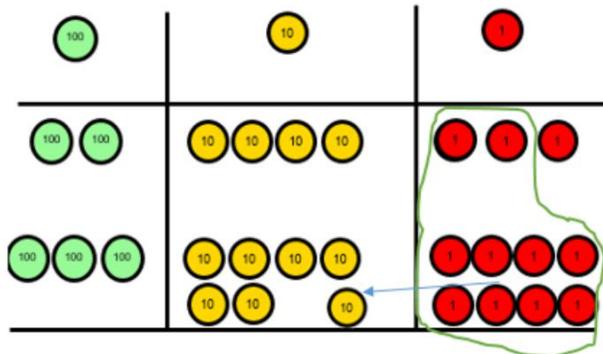
Looking for ways to make 10



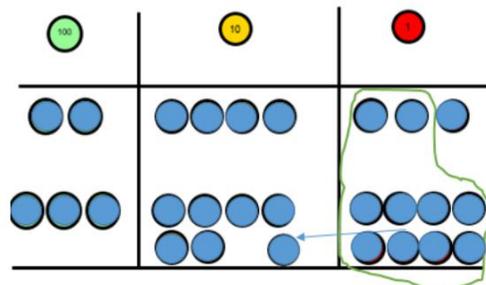
Formal method:

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline 1 \end{array}$$

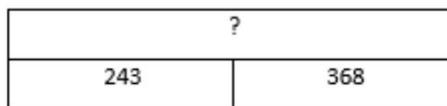
Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Children to represent the counters e.g. like the image below

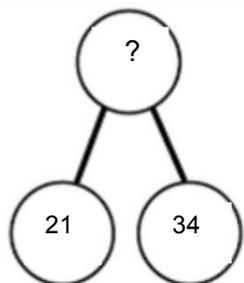


If the children are completing a word problem, draw a bar model to represent what it's asking them to do



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 11 \end{array}$$

## Fluency variation, different ways to ask children to solve 21+34:



Sam saved £21 one week and £34 another. How much did he save in total?

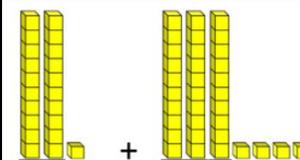
$21+34=55$ . Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\square = 21 + 34$$

What's the sum of twenty one and thirty four?

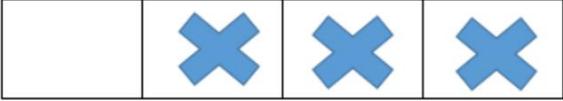
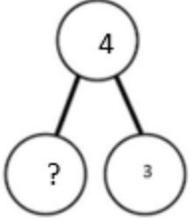
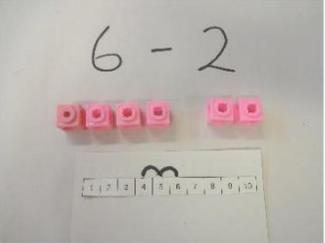
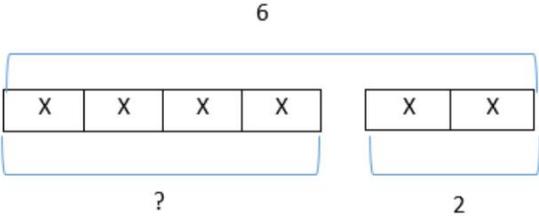
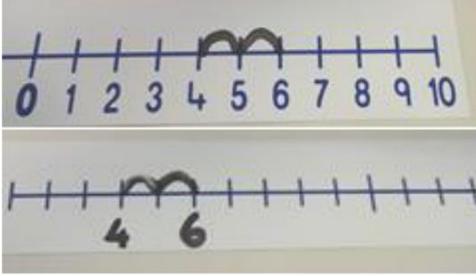


Always use missing digit problems too:

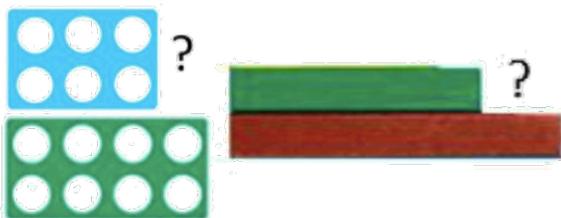
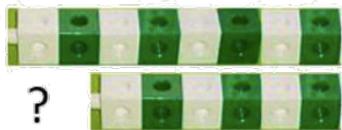
Tens	Ones
	?
?	4

# Subtraction-

Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'

Concrete	Pictorial	Abstract
<p><b>Physically taking away and removing objects from a whole</b> (use various objects too) rather than crossing out-children will physically remove the objects</p> <p><math>4 - 3 = 1</math></p>  <p>The image shows four green plastic cups on a green tray, each with a blue 'X' drawn on it. To the right of the cups is a row of four wooden blocks, also with blue 'X' marks on their top surfaces.</p>	<p>Children to draw the concrete resources they are using and cross out.</p>  <p>The image shows four simple drawings of birds in a row. The first bird is green, and the other three are purple. Each of the three purple birds has a large blue 'X' drawn over its body.</p> <p>Use of the bar model:</p>  <p>A horizontal bar divided into four equal sections. The first section is empty, and the next three sections each contain a large blue 'X'.</p>	<p><math>4 - 3 =</math></p> <p><input type="text"/> = <math>4 - 3</math></p>  <p>A horizontal bar divided into three equal sections. The top section is labeled '4', the bottom-left section is labeled '3', and the bottom-right section is labeled '?'.</p>  <p>A number bond diagram with a large circle at the top containing the number '4'. Two lines connect it to two smaller circles below. The left circle contains a question mark '?' and the right circle contains the number '3'.</p>
<p><b>Counting back</b> (using number lines or number tracks)</p>  <p>The image shows the equation <math>6 - 2</math> written on a whiteboard. Below it are six pink counting blocks arranged in a row. A number line is visible at the bottom, with a bracket indicating a jump back from 6 to 4.</p>	<p>Children to represent what they see pictorially e.g.</p>  <p>A bar model for the equation <math>6 - 2 = ?</math>. The top of the bar is labeled '6'. The bar is divided into six equal sections. The first four sections each contain an 'X', and the last two sections each contain an 'X'. Below the bar, a bracket under the first four sections is labeled '?', and a bracket under the last two sections is labeled '2'.</p>	 <p>Two number lines are shown. The top one is a number line from 0 to 10 with tick marks at every integer. A bracket is drawn above the line from 6 to 4, with a curved arrow pointing from 6 to 4. The bottom one is a similar number line with a bracket drawn below the line from 6 to 4, with a curved arrow pointing from 6 to 4.</p>

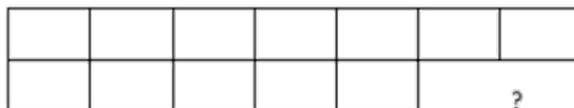
**Finding the difference** (using cubes, numicon or Cuisenaire rods, other objects can also be used)



Children to draw the cubes/other concrete objects which they have used

XXXXXXXXX  
XXXXXX

Use of the bar model

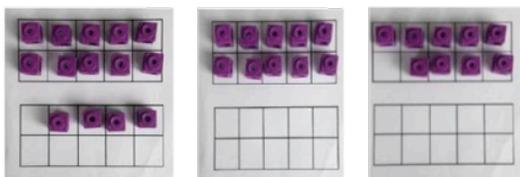


Find the difference between 8 and 6.

$8 - 6$ , the difference is ?

Children to also explore why  $9 - 7 = 8 - 6$  (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)

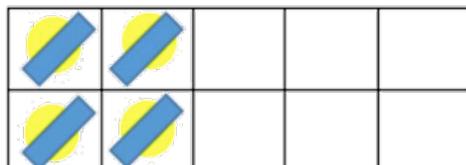
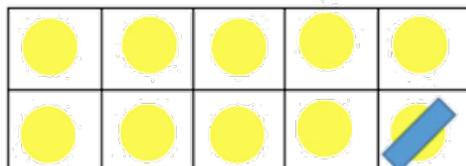
**Making 10** (using numicon or ten frames)  
 $14 - 5$



Children could also do this by subtracting a 5 from the 10.

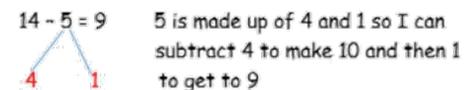
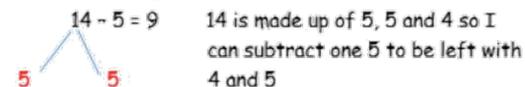


Children to present the ten frame pictorially

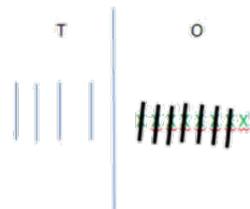
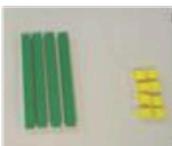


$14 - 5 = 9$  You also want children to see related facts e.g.  $15 - 9 = 5$

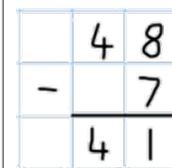
Children to represent how they have solved it e.g.



**Column method** (using base 10)  
 $48 - 7$

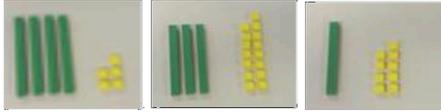


$48 - 7 =$



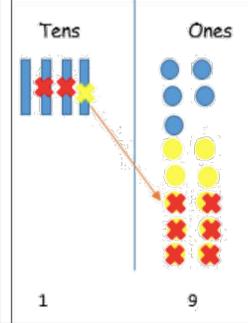
**Column method** (using base 10 and having to exchange)

45-26

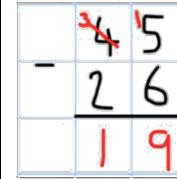


- 1) Start by partitioning 45
- 2) Exchange one ten for ten more ones
- 3) Subtract the ones, then the tens.

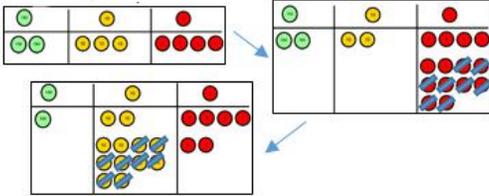
**Represent the base 10 pictorially**



It's crucial that the children understand that when they have exchanged the 10 they still have 45.  $45 = 30 + 15$

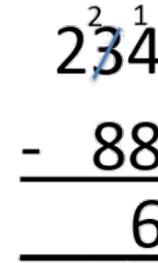


**Column method** (using place value counters) 234-88

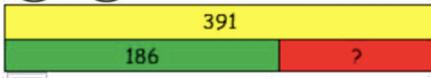
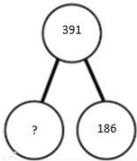


Once the children have had practice with the concrete, they should be able to apply it to any subtraction.

Like the other pictorial representations, children to represent the counters.



## Fluency variation, different ways to ask children to solve 391-186:



Raj spent £391, Timmy spent £186. How much more did Raj spend?

I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

$$391 - 186$$

$$\square = 391 - 186$$

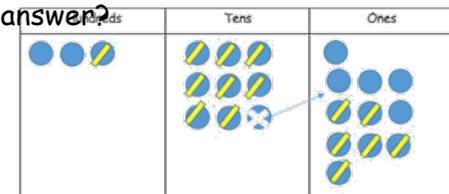
$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

Find the difference between 391 and 186

Subtract 186 from 391.

What is 186 less than 391?

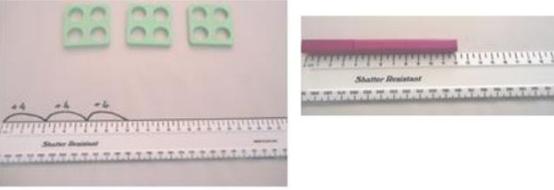
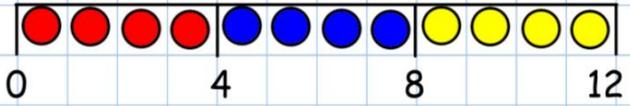
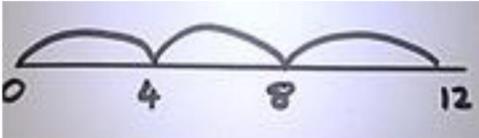
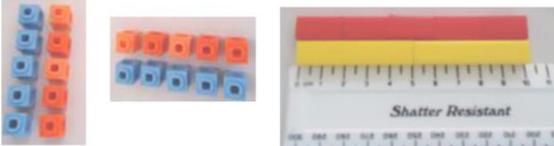
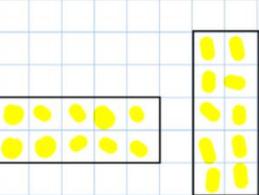
What's the calculation? What's the answer?



$$\begin{array}{r} 391 \\ -186 \\ \hline 205 \end{array}$$

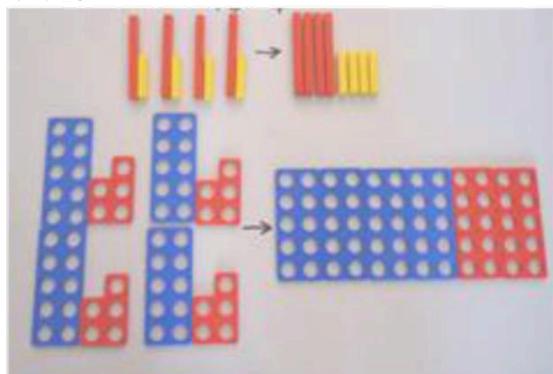
# Multiplication-

Key language which should be used: double times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as'

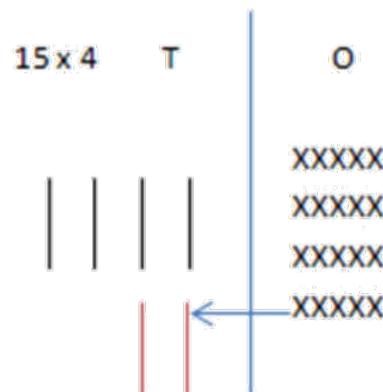
Concrete	Pictorial	Abstract
<p><b>Repeated grouping/repeated addition</b> (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4</p> 	<p>Children to represent the practical resources in a picture e.g.</p> <p>XX XX XX XX XX XX</p> <p>Use of a bar model for a more structured method</p> 	<p>3 x 4</p> <p>4 + 4 + 4</p>
<p><b>Use number lines to show repeated groups- 3 x 4</b></p> 	<p>Represent this pictorially alongside a number line e.g:</p> 	<p>Abstract number line</p> <p>3 x 4 = 12</p> 
<p><b>Use arrays to illustrate commutativity</b> (counters and other objects can also be used) 2 x 5 = 5 x 2</p> 	<p>Children to draw the arrays</p> 	<p>Children to be able to use an array to write a range of calculations e.g.</p> <p>2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 = 10</p>

**Partition to multiply** (use numicon, base 10, Cuisenaire rods)

$4 \times 15$



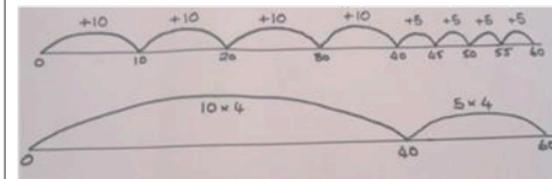
Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like:



Children to be encouraged to show the steps they have taken

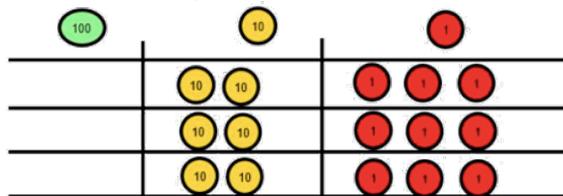
$$\begin{array}{r}
 4 \times 15 \\
 \swarrow \searrow \\
 10 \quad 5 \\
 \\
 10 \times 4 = 40 \\
 5 \times 4 = 20 \\
 40 + 20 = 60
 \end{array}$$

A number line can also be used



**Formal column method** with place value counters or base 10 (at the first stage- no exchanging)  $3 \times 23$

Make 23, 3 times. See how many ones, then how many tens



Children to represent the counters in a pictorial way

Tens	Ones
6	9

Children to record what it is they are doing to show understanding

$$\begin{array}{r}
 3 \times 23 \\
 \swarrow \searrow \\
 20 \quad 3
 \end{array}
 \quad
 \begin{array}{r}
 3 \times 20 = 60 \\
 3 \times 3 = 9 \\
 60 + 9 = 69
 \end{array}$$

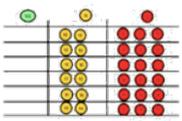
$$\begin{array}{r}
 23 \\
 \times 3 \\
 \hline
 69
 \end{array}$$

**Formal column method** with place value counters (children need this stage, initially, to understand how the column method works)

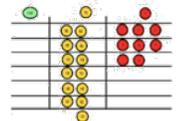
Children to represent the counters/base 10, pictorially e.g. the image below.

$$\begin{array}{r}
 6 \times 23 \\
 6 \times 3 = 18 \\
 6 \times 20 = 120 \\
 120 + 18 = 138
 \end{array}$$

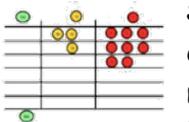
$6 \times 23$



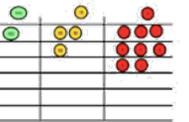
**Step 1:** get 6 lots of 23



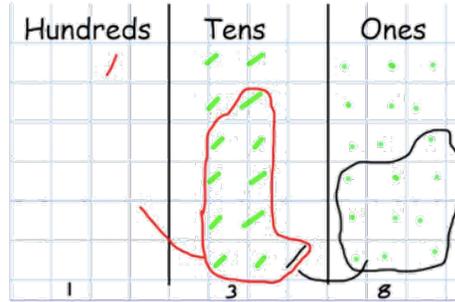
**Step 2:**  $6 \times 3$  is 18. Can I make an exchange? Yes! Ten ones for one ten....



**Step 3:**  $6 \times 2$  tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...



**Step 4-** what do I have in each column?



The aim is to get to the formal method but the children need to understand how it works.

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 11
 \end{array}$$

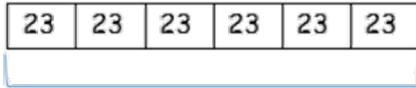
When children start to multiply  $3d \times 3d$  and  $4d \times 2d$  etc, they should be confident with the abstract:

To get 744 children have solved  $6 \times 124$   
 To get 2480 they have solved  $20 \times 124$

$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 1120 \\
 \hline
 3224 \\
 \hline
 11
 \end{array}$$

Answer: 3224

# Fluency variation, different ways to ask children to solve $6 \times 23$ :



?

With the counters, prove that  $6 \times 23 = 138$

Why is  $6 \times 23 = 32 \times 6$ ?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

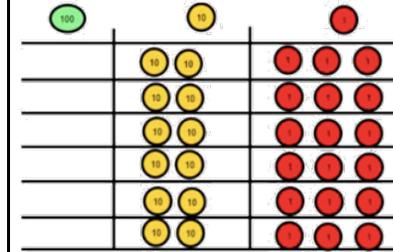
Tom saved 23p three days a week. How much did he save in 2 weeks?

Find the product of 6 and 23

$$6 \times 23 =$$

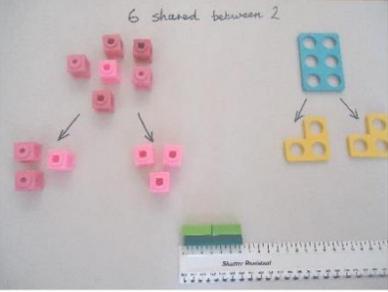
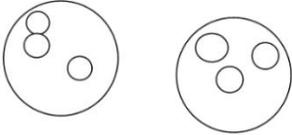
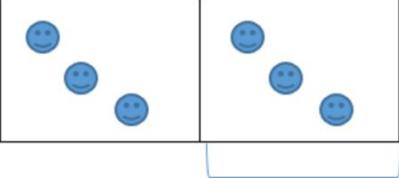
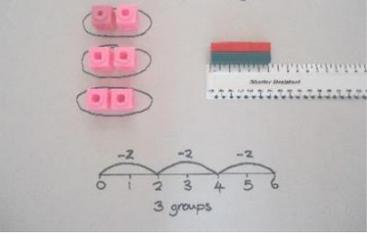
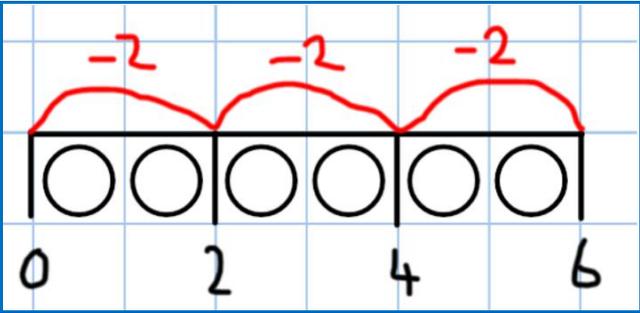
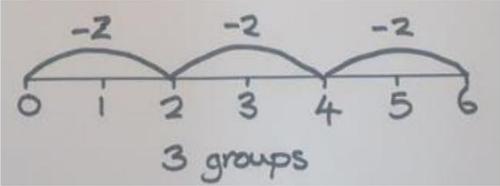
$$\begin{array}{r} \square = 6 \times 23 \\ 6 \quad 23 \\ \times \underline{23} \quad \times \underline{6} \\ \hline \hline \end{array}$$

What's the calculation? What's the answer?

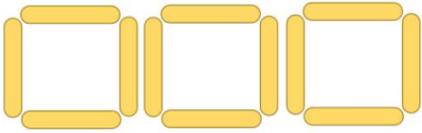


# Division-

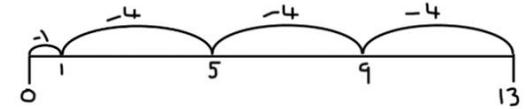
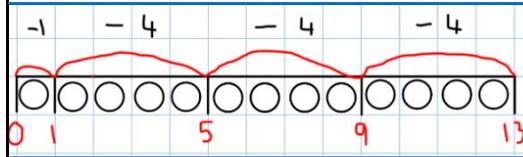
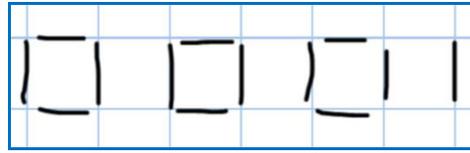
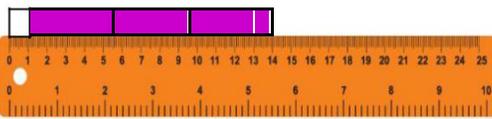
Key language which should be used: share, group, divide, divided by, half, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract		
<p><b>6 shared between 2</b> (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)</p>  <p>6 shared between 2</p>	 <p>This can also be done in a bar so all 4 operations have a similar structure:</p> 	<p><math>6 \div 2 = 3</math></p> <p>What's the calculation?</p> <table border="1" data-bbox="1402 548 1850 613"> <tr> <td>3</td> <td>3</td> </tr> </table>	3	3
3	3			
<p><b>Understand division as repeated grouping and subtracting</b></p> <p><math>6 \div 2</math></p>  <p>3 groups</p>		<p>Abstract number line</p>  <p>3 groups</p>		
<p><b>2d ÷ 1d with remainders</b></p> <p><math>13 \div 4 = 3</math> remainder 1</p>	<p>Children to have chance to represent the resources they use in a pictorial way e.g. see below:</p>	<p><math>13 \div 4 = 3</math> remainder 1</p> <p>Children to count their times tables facts in their heads</p>		

Use of lollipop sticks to form wholes

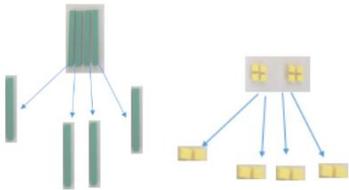


Use of Cuisenaire rods and rulers (using repeated subtraction)



2d divided by 1d using base 10 (no remainders) SHARING

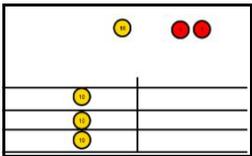
$$48 \div 4 = 12$$



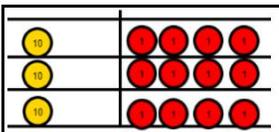
Start with the tens.

Sharing using place value counters.

$$42 \div 3 = 14$$

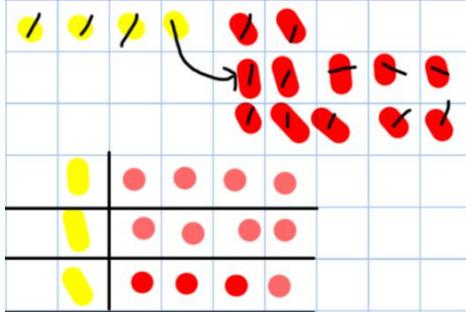


1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?



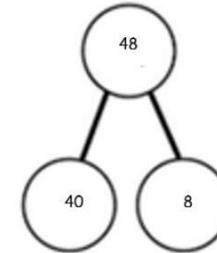
Exchange the ten for 10 ones and share out 12 ones

Children to represent the base 10 and sharing pictorially.



$$48 \div 4$$

4 tens  $\div$  4 = 1 ten  
8 ones  $\div$  4 = 2 ones



$$10 + 2 = 12$$

$$42 \div 3$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

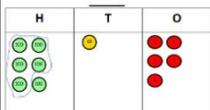
$$10 + 4 = 14$$

Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- *this can also be done using sharing!*

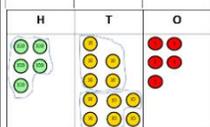
$$615 \div 5$$



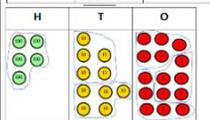
Step 1: make 615



Step 2: Circle your groups of 5



Step 3: Exchange 1H for 10T and circle groups of 5



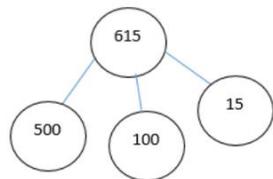
Step 4: exchange 1T for 10ones and circles groups of 5

This can easily be represented pictorially, till the children no longer to do it. It can also be done to decimal places if you have a remainder!

$$5 \overline{) 615} \begin{matrix} 123 \\ \hline \end{matrix}$$

## Fluency variation, different ways to ask children to solve $615 \div 5$ :

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

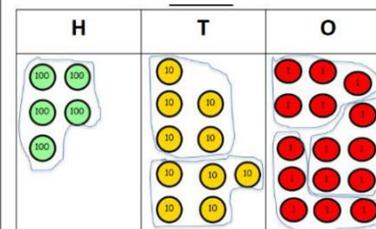
$$5 \overline{) 615}$$

$$615 \div 5 =$$

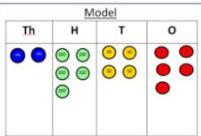
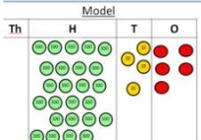
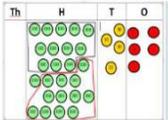
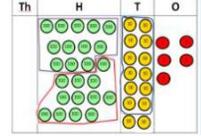
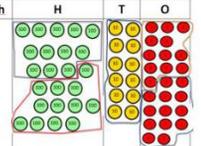
$$\square = 615 \div 5$$

How many 5's go into 615?

What's the calculation? What's the answer?



## Long division

Concrete	Pictorial	Abstract
 <p><math>2544 \div 12</math>  <math display="block">\begin{array}{r} 0212 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}</math></p> <p>How many groups of 12 thousands do we have? None</p>	<p>Children to represent the counters, pictorially and record the subtractions beneath.</p>	<p>Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.</p> $12 \overline{)2544}^0$
 <p>Exchange 2 thousand for 20 hundreds.</p>		<p>Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.</p> $12 \overline{)2544}^{02}$
 <p><math>12 \overline{)2544}^{02}</math>  <math display="block">\begin{array}{r} 02 \\ 12 \overline{)2544} \\ \underline{24} \\ 1 \end{array}</math></p> <p>How many groups of 12 are in 25 hundreds? 2 groups. Circle them.</p>		<p>Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens?</p> $12 \overline{)2544}^{021}$
<p>We have grouped 24 hundreds so can take them off and we are left with one.</p>		<p>The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.</p> $12 \overline{)2544}^{0212}$
 <p><math>12 \overline{)2544}^{021}</math>  <math display="block">\begin{array}{r} 021 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}</math></p> <p>Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.</p>		<p>Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.</p> $12 \overline{)2544}^{0212}$
 <p>Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2</p>		$12 \overline{)2544}^{0212}$